

Some (Mostly Unoriginal) Games and Puzzles

This is a collection of games and puzzles with very few prerequisites to get started. They can be adapted to fit many grade levels and knowledge, and be done in groups of many different sizes.

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Contents

1	Sprout	2
2	See-It-Say-It	4
3	Magic Squares	5
4	Three House Puzzle	6
5	Connect Four	7
6	Bulls and Cows	8
7	21 and Over	10
8	Factor Secret Message	12

Chapter 1

Sprout

A game requiring 2 or more people.

Supplies: A piece of paper, writing utensils for each player. They can, but don't need to be, different colors.

Rules:

- Start by placing some dots on the paper. It doesn't really matter where or how many, but 4 to 8 is a good way to start.
- On each turn, a player draws a line from one dot to another. Then, they place a new dot somewhere in the middle of their line.
- Lines *cannot cross*.
- Each dot can have at most *three lines* coming out from it. Once a dot has three lines emerging from it, cross it out and do not use it.
- The last player to draw a line wins.

Sample Game:

We'll show a few steps of a game between Black and Orange, using 2 dots.



Black will go first. They draw a line, and place a new dot on their line. Notice that Black's new dot counts as having *two* lines coming from it.



Then, Orange will go.



Then Black goes. They connect a dot that includes the dot Orange just made, so that dot

is no longer available! Also, the right starting dot is unavailable. We make them large and red to make this clear. We continue in this way.



Given the current arrangement of the game, Orange goes next. Can Orange guarantee a win for themselves at all? How will this game end?

Solution: The current arrangement of the game is symmetric. The three remaining dots each have two lines coming out of them. Orange can guarantee a win by connecting the two bottom-left dots *inside* the triangle. Then they will add a dot on their line, but it cannot be connected to the remaining dot in the top-right. Any other move guarantees a Black victory.

Presentation Ideas:

- Change the rules! What happens if you allow lines to cross, and increase how many lines can go through a dot?
- Present these as puzzles. Create some nearly-completed games, and present the challenge to find the available winning move.
- How does your strategy change when there are more than 2 people playing?

Chapter 2

See-It-Say-It

This is a puzzle where students try to determine the rule for the following sequence:

1
11
21
1211
111221
312211
⋮

The Pattern: The sequence is called “See-It-Say-It” because that’s how it is constructed. We start with 1. The first term has “one 1”, so we write down 11 for the second term. The second term has “two 1’s”, so we write down 21 for the third term. The third term has “one 2, one 1”, so we write down 12 11 \implies 1211 for the fourth term. (Spaces represent a comma in the phrase). The fourth term has “one 1, one 2, and two 1’s”, so we write 11 12 21 \implies 111221 for the fifth term. And so on.

Presentation Ideas:

- Typically refrain from telling them what the sequence is called. Start by putting the first three numbers down, and ask for the next term. Nearly every student will put 31.
- Surprise them by writing 1211 instead. (I don’t use commas because that makes spotting the pattern even more difficult.) Ask them again to work on finding the next term.
- If they find the rule, or you decide to tell them, have them try to spot patterns or come up with their own variations.
- Challenge them to find a term with a digit greater than 3 in it (it will never happen.)

Chapter 3

Magic Squares

A *Magic Square* is a square grid of numbers where every row, column, and diagonal sums to the same number. A true magic square will have all unique numbers. Here's the traditional 3×3 magic square using the numbers 1 to 9, where the sum everywhere is 15:

4	9	2
3	5	7
8	1	6

Presentation Ideas: The first thing to do is challenge students to get the magic square above (of course it can be rotated/reflected). Give them the digits they are allowed to use; to make it a bit easier, also give them the target sum of 15.

An extra option is to “challenge” them to find a 2×2 magic square where every number is different. It's impossible for pretty trivial reasons (if you fix the top-left corner, the squares to its right and below it *must* be equal), but it's good for them to come to that conclusion themselves.

Some other magic squares:

- 3×3 using 7, 8, 9, 12, 13, 14, 17, 18, 19. (Sum is 39.)
- 4×4 using 1 through 16. (Sum is 34.)
- 3×3 using 11, 17, 19, 23, 25, 27, 31, 33, 39. (Sum is 75.)

Also, you can challenge them to find a 3×3 magic square where each entry is a unique perfect square. There is no known solution to that puzzle.

Chapter 4

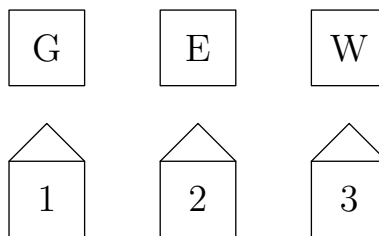
Three House Puzzle

Disclaimer: This puzzle is impossible with the assumed rules of using a single side of a sheet of paper. But more on that later.

Supplies: Paper and pencil.

Rules: There are three houses. They each need gas, electricity, and water. However, these “pipes” cannot cross. Can you connect each of the three utilities to each house without any lines crossing?

The typical diagram is given below.



Presentation Ideas:

- Of course, you can just let them sit there frustrated. If they claim to have a solution on their single side of paper, you know there’s an issue.
- Loosen the restrictions. Make up stuff about “wormholes” (or whatever the alternate reality was in *Stranger Things*) to explain they can now try using the back of the paper! Suddenly it’s possible.
- What happens if you cut a hole in the middle of the paper to make it a donut?
- Depending on what level of math they can understand, you can throw words out there like “torus”. Otherwise, you can let them know it’s possible to do on a coffee mug.

Chapter 5

Connect Four

Surprisingly easy to turn into a paper-and-pencil game using graph paper. If nothing else, it's a good strategy game to keep kids mentally engaged.

Supplies: Paper and pencil. Colors are fine, or students can play as \times and \circ .

Rules: The standard game board is 6 holes deep by 7 holes wide. Students take turns “dropping” pieces (they just need to imagine gravity exists on their paper) into the board. The first to make four in a row wins.

Presentation Ideas:

- After playing for a little bit and developing a strategy, change the size of the game board.
- Grab another color or make up another symbol, and try playing with more than 2 people. How does that change how you play? Everybody has 2 other people to try and defend against.
- Remove “gravity” from the game, and turn it into a larger version of tic-tac-toe. How does that work?
- Other possible board variations include putting a “free space” somewhere that either player can use; remove one (or two, or three) spaces in random places; with or without gravity, change the *shape* of the game board, making a rhombus or triangle or something to that effect.

Chapter 6

Bulls and Cows

If you've played the board game *Mastermind*, this is the same thing.

Supplies: Paper and pencil. It's better for effect if you have some scraps of paper to write the target number down on.

Rules:

- The short version: One player (P1) picks a (3, 4, 5, 6 digit number), the other player (P2) must try to guess it. I typically play without allowing 0 as a digit.
- Logistically, P1 should write it down somewhere hidden where they can reference it. Then, P2 should write down each guess on another sheet of paper.
- On each guess, P1 gives two pieces of information: How many correct digits are in the correct place, and how many correct digits are in the wrong place.
- Visually, this is written down using • and ○. The former is correct digits in correct places, the latter is correct digits in the wrong place.
- Traditionally, P1 chooses a 4-digit number, and P2 has 10 guesses. If they don't get it after 10 guesses, then P1 wins. It's proven every 4-digit number can be guessed with 7 chances, so you can make reduce the number of guesses to make it more stressful.
- For other lengths of numbers, I'm unsure how many turns are necessary, but you could simply play two rounds and the player who guesses a number in fewer turns wins.

Let's suppose P1 chooses 1834, and P2 guesses 4823. Then P2 has a 4 and 3 in the incorrect spot, and an 8 in the correct spot. P1 would convey this information as shown:

4823 | • ○ ○

and P2 would then revise their guess in whatever way they see fit. The order in which P1 writes • and ○ is immaterial.

Presentation Ideas:

- You can just play the game!
- Try to come up with a good strategy.

- Come up with *puzzles*, where you have a string of guesses from which it is possible to find the chosen number. See below for a couple of examples.

Sample Puzzles:

1. Try to find the number I chose, given the information below.

137	•
196	• ○
165	○

Solution: In our first guess, we have one number in the correct place. We guess that it's 1, which seems confirmed by the result of our second guess. However, we keep a 1 in the first place for our third guess, and there is now no •. In other words, 1 is not in the chosen number.

Now, we know the number has a 3 or 7 from the first guess. The second guess tells us 9 and 6 are both in the chosen number, and one of them is in the correct place. So, the 5 in the third guess is immaterial, and we know that moving the 6 makes it in the wrong place. So, the 6 is in the third spot, based on the second guess.

This means the 9 in the second place is wrong, and it must go in the first place. We have the first and third places locked down, which means the 3 in the first guess must be the correct number in the correct place. Hence, I chose 936 as my number.

2. Find two possibilities for the number I chose, given the information below.

1368	○
2154	• • ○
2645	• •
7145	• • ○

Solution: We'll see what information we can determine for certain in this situation, then construct whatever examples we can. The first guess gives us four possibilities for a digit in our number. The second and third guesses combine to guarantee there is 1 in the number: the only digit we replace is a 1 for 6, and the number of correct digits at all reduce by 1. We conclude there is a 1, and from the first guess it is not in the first spot.

Now we get into the tricky part. First, we claim the number must have a 2. If not, based on guesses 2 and 4, three of the digits must be 1, 4, and 5. Since 2, 3, 6, 7, and 8 are out of contention now, the fourth digit must be 9. However, there is no way to arrange 9, 1, 4, and 5 in a way consistent with the guesses (go ahead and try.) Therefore, 2 must be a digit in the number.

Now that we've shown 2 is in the number, the third guess makes it clear that 4 and 5 cannot *both* be in the number. The result of this is that the third guess also tells us the 2 must be in the first spot. Also, the fourth guess implies the final digit must be 7. So, the digits are 1, 2, (4 or 5), and 7. We know 2 is in the first spot now, which means 7 is the number in the wrong spot in the fourth guess. Since only one of 4 and 5 is in the number, we just replace the other with a 7. So, 2175 and 2147 are the two possible numbers.

Chapter 7

21 and Over

The only game here that's fully my invention. I made it for teaching mixed number addition in 4th grade math. It's probably a bit more fun when you actually know mixed numbers, but reports are fairly fine.

Supplies: Pencil and paper, and two standard dice (or sufficient number generator.)

Rules:

- Roll two dice and add them together. This is the *denominator* of every mixed number you make.
- Each player rolls one die. The person who rolls the biggest number goes first. Each player starts with a total of 0.
- On your turn:
 - Roll both dice. Choose one die to be the whole part of your mixed number and one die to be the numerator of your mixed number.
 - Write down this mixed number and add it to your total.
 - Let the other players check your addition, and fix it if you made an error.
- If you are close to 21, you can choose to stop. The other players can still try to get closer to 21.
- If you go over 21, you are out of the game.
- The player closest to 21 who does not go over 21 wins!

Sample Game: Grogg and Winnie play a game of **21 And Over**. When they roll the dice to start, they roll a 5 and 1. So, the denominator of every mixed number is **6**.

Grogg rolls a 3 and Winnie rolls a 5, so Winnie goes first.

Winnie rolls a 3 and a 4. She makes the mixed number $4\frac{3}{6}$. She writes it in a table like the one below. Her total is $4\frac{3}{6}$.

Grogg rolls a 5 and a 6. He makes the mixed number $5\frac{6}{6}$. He writes it in the table as 6. His total is 6.

Winnie's Numbers	Winnie's Total	Grogg's Numbers	Grogg's Total
$4\frac{3}{6}$	$4\frac{3}{6}$	6	6

The next few rounds are shown in the table.

Winnie's Numbers	Winnie's Total	Grogg's Numbers	Grogg's Total
$4\frac{3}{6}$	$4\frac{3}{6}$	6	6
$5\frac{2}{6}$	$9\frac{5}{6}$	$5\frac{5}{6}$	$11\frac{5}{6}$
$4\frac{1}{6}$	14	$2\frac{1}{3}$	$14\frac{1}{6}$
$3\frac{2}{6}$	$17\frac{2}{6}$	$4\frac{1}{6}$	$18\frac{2}{6}$
$2\frac{4}{6}$	20		

Winnie decides to stop at 20.

Grogg is at $18\frac{2}{6}$, so he rolls again. He gets a 2 and a 4. He picks the number $2\frac{4}{6}$. His new total is $18\frac{2}{6} + 2\frac{4}{6} = 21$. Since Grogg got closer to 21 without going over 21, **Grogg wins**.

Presentation Ideas: Nothing immediately comes to mind, but you can always increase the number you're playing to. The first iteration went to 50, until it was decided that probably wasn't the best for fourth graders.

Chapter 8

Factor Secret Message

This is another puzzle I devised for fourth graders. It probably stands on its own fairly well for your casual person as well. It gets reasonably subtle at the end. Good luck.

The Puzzle: Find the value of *SECRET* using the clues below. Go to the next page for the solution.

C, *E*, *R*, *S*, and *T* are digits from 0 to 9. You can find them with the following clues:

1. *S* is divisible by 7.
2. *SE* is divisible by 6.
3. *SEC* is divisible by 5 and 2.
4. *SECR* is divisible by 4.
5. *SECRE* is divisible by 3.
6. *SECRET* is divisible by 2.
7. *SECRETS* is divisible by 9.

Problem: $C, E, R, S,$ and T are digits from 0 to 9. You can find them with the following clues:

1. S is divisible by 7.
2. SE is divisible by 6.
3. SEC is divisible by 5 and 2.
4. $SECR$ is divisible by 4.
5. $SECRE$ is divisible by 3.
6. $SECRET$ is divisible by 2.
7. $SECRETS$ is divisible by 9.

Solution: We start with the first clue: S is divisible by 7. Since each number has S as the first digit, and a number cannot start with 0, this clue tells us $S = 7$.

The second clue tells us either $E = 2$ or $E = 8$.

The third clue tells us SEC is divisible by 10, and hence $C = 0$.

The fourth clue tells us CR must be divisible by 4. Since $C = 0$, this means either $R = 4$ or $R = 8$.

The fifth clue tells us $S + E + C + R + E$ is divisible by 3. Since $S = 7$ and $C = 0$, we know $7 + E + R + E$ must be divisible by 3. There are four cases, since E is 2 or 8, and R is 4 or 8.

E	R	$7 + E + R + E$
2	4	15
8	4	27
2	8	19
8	8	31

Since $7 + E + R + E$ is only divisible by 3 when $R = 4$, we know $R = 4$.

The sixth clue tells us T is an even digit.

The seventh clue tells us $S + E + C + R + E + T + S$ is divisible by 9. Filling in $S = 7, C = 0,$ and $R = 4,$ this means $18 + E + E + T$ must be divisible by 9.

If $E = 2,$ this means $22 + T$ must be divisible by 9. But there is no even digit T that makes this true. So, we know $E = 8$.

Then, we have $18 + E + E + T = 18 + 8 + 8 + T = 34 + T$ is divisible by 9. The only possible option is $T = 2$.

Therefore, $SECRET = 780482$.